PREDICTION OF ERODED VERSUS ACCRETED BEACHES

PURPOSE: To present revised procedures for predicting whether a beach profile of a specified sand size will tend to erode or accrete under incident waves of given height and period. This CETN supplements pages 4-83 through 4-85 of the Shore Protection Manual (SPM 1984) which describe beach change produced by cross-shore sand transport.

BACKGROUND: The term "erosion" usually describes subaerial erosion, i.e., removal of material from the visible beach, often to produce a gentle slope in the surf zone and one or more large longshore bars in the offshore. The term "accretion" usually describes sand accumulation in the form of one or more berms on the visible beach and, typically, a steep profile in the surf zone with relatively small offshore bars. Although erosion and accretion commonly refer to the response of the subaerial beach, material may not be lost or gained in the total system, but only displaced and rearranged. Surveys of wide longshore and cross-shore extent are needed to determine if a beach has actually lost or gained material.

In the early days of coastal engineering, it was thought that wave steepness was a sufficient predictor of erosion or accretion, with waves of large steepness producing erosion and small steepness producing accretion. It was later realized that the height of the waves was also important, as well as the median grain size of the beach; fine-sand beaches typically undergo greater variations than coarse-sand beaches. For predominantly quartz sand beaches, a sieve-determined median diameter is probably an adequate descriptor of grain size. For beaches comprised of particles of different densities and shapes, however, or to account for temperature effects (water viscosity), the sediment particle fall velocity is believed to be a more valid representation of "hydraulic grain size." Fall velocity may be calculated by Equations 4-7 - 4-9 of the SPM (1984) (see also, CETN II-4).

Laboratory and field measurements have indicated that the following variables determine in great part whether a beach will erode or accrete:

- wave steepness: \( H_0 / L_0 \)
- wave height: \( H_0 \)
- median grain size: \( d_{50} \) (or, equivalently, sand fall velocity, \( w \))

In the above, wave height \( H \) and wavelength \( L \) are evaluated in deep water, denoted by the subscript "o." The deepwater wavelength is given by linear wave theory as \( L_0 = \frac{gT^2}{2\pi} \), where \( g \) is the acceleration of gravity (\( g = 9.81 \text{ m/sec}^2 = 32.2 \text{ ft/sec}^2 \)), and \( T \) is the wave period. In metric units (m), \( L_0 = 1.56 T^2 \), whereas in American Customary units (ft), \( L_0 = 5.12 T^2 \), for which \( T \) is given in sec. Thus, wave period is also a factor controlling beach erosion and accretion.

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The predictive equation of Sunamura and Horikawa (1975), appearing on page 4-85 of the SPM (1984), contains the average beach slope \( \tan \beta \) in addition to the parameters mentioned above. This predictive criterion is given by

\[
C = \frac{H_o}{L_o} (\tan \beta)^{0.27} \left( \frac{d}{L_o} \right)^{-0.67}
\]

\( C > 18 \), erosion
\( 9 \leq C \leq 18 \), undetermined or mixed
\( C < 9 \), accretion

(Note: Equation 1 replaces the version given in the SPM (1984), which contains typographic errors)

The average beach slope may be unavailable or difficult to determine; also, since beach slope is correlated with grain size, inclusion of both parameters in one predictor may be an overspecification. Nevertheless, Eq. 1 can still be used since it was calibrated with field data.

**Prediction Methods**: Three criteria are presented for estimating whether a beach of known median grain size will erode or accrete due to cross-shore transport produced by incident waves of specified characteristics. These criteria were developed based on two sets of laboratory data (labeled CE and CRIEPI in figures given below) involving wave and beach dimensions of prototype scale and monochromatic waves (Larson and Kraus 1988). In evaluation of a representative wave height and period in applications, significant wave height and peak spectral period should be used. Similar predictive criteria, based on model and prototype size laboratory conditions, were presented by Kohler and Galvin (1973) and appeared in the 3rd edition of the SPM (1977).

**Criterion 1**: This criterion uses the deepwater wave steepness and dimensionless fall velocity, defined as \( H_o/wT \), and is expressed as

\[
\frac{H_o}{L_o} \leq 0.00070 \left( \frac{H_o}{wT} \right)^3, \quad \text{erosion}
\]

\[
\frac{H_o}{L_o} > 0.00070 \left( \frac{H_o}{wT} \right)^3, \quad \text{accretion}
\]

This criterion is shown in Fig. 1 with the data upon which it is based. Wave steepness and nondimensional fall velocity combinations producing a prominent berm are labeled with open symbols, and combinations giving a prominent bar are labeled with filled symbols. The diagonal line expressing the criterion separates regions occupied by open symbols (accretion) and closed symbols (erosion). Eq. 2 extends the discussion of the fall velocity parameter given on page 4-85 of the SPM (1984) by incorporating wave steepness in prediction of erosion and accretion.
Criterion 2: This criterion is expressed in terms of the deepwater wave steepness and the ratio of deepwater wave height and median grain size, as shown by the diagonal line in Fig. 2, and is given by

\[
\frac{H_o}{L_o} < 4.8 \times 10^8 \left( \frac{H_o}{d_{50}} \right)^{-3.05}, \text{ accretion}
\]

\[
\frac{H_o}{L_o} \leq 4.8 \times 10^8 \left( \frac{H_o}{d_{50}} \right)^{-3.05}, \text{ erosion}
\]

Eq. 3 is easy to apply since it is expressed in terms of readily available variables; however, it is strictly limited to quartz sand and water temperatures well above freezing.

Criterion 3: This criterion is expressed in terms of deepwater wave steepness and Dean's (1973) parameter \( \pi w/gT \), which is formed with the grain fall velocity and wave period. The criterion is expressed by

\[
\frac{H_o}{L_o} < 115 \left( \frac{\pi w}{gT} \right)^{1.5}, \text{ accretion}
\]

\[
\frac{H_o}{L_o} \geq 115 \left( \frac{\pi w}{gT} \right)^{1.5}, \text{ erosion}
\]

Eq. 4 is plotted as line C in Fig. 3. Line A expresses a criterion originally given by Dean (1973) which is based on erosion and accretion produced in small wave tanks; line B lies parallel to line A with a coefficient re-evaluated for prototype-scale data. The change in coefficient value represents a scale effect, since sand transport processes are different under small and large waves, even if steepness is the same. Although line B gives an improvement over line A, the rotated line C defined by Eq. 4 provides a better separation of all erosional and accretionary cases.

DISCUSSION: The three criteria defined by Equations 2, 3, and 4 for predicting whether a beach will tend to erode or accrete were developed from the same data base. Therefore, empirically the criteria are equivalent. From a theoretical viewpoint, Eq. 2, which incorporates the magnitude of wave height and grain fall velocity, is superior. However, for applications involving quartz beach sands and typical water temperatures, any of the three criteria may be applied. Finally, it is emphasized that the material and procedures described here pertain to beach change produced by cross-shore sand transport under wave action. Erosion and accretion may also be caused by longshore sand transport-related processes.
Figure 1. Criterion for distinguishing bar and berm profiles by use of wave steepness and dimensionless fall velocity (Larson and Kraus 1988)

Figure 2. Criterion for distinguishing bar and berm profiles by use of wave steepness and ratio of wave height to grain size (Larson and Kraus 1988)
Figure 3. Criterion for distinguishing bar and berm profiles by use of deepwater wave steepness and Dean's parameter (Larson and Kraus 1988)

Two examples, distinguished by different median grain sizes, are given to illustrate use of the above criteria for predicting beach erosion and accretion caused by cross-shore transport.

**EXAMPLES**

Given:

- **A** quartz sand
  - \(d_{50} = 0.2 \text{ mm}\)
  - \(H_0 = 1 \text{ m}\)
  - \(T = 10 \text{ sec}\)

- **B** quartz sand
  - \(d_{50} = 0.4 \text{ mm}\)
  - \(H_0 = 1 \text{ m}\)
  - \(T = 10 \text{ sec}\)

**Problem:** Determine, using the criteria presented, whether the beach will experience erosion or accretion. [Required constants: density of quartz sand, \(\rho_s = 2.65 \text{ g/cm}^3\); density of seawater at 20°C, \(\rho = 1.025 \text{ g/cm}^3\); and kinematic viscosity of seawater at 20°C, \(\nu = 0.01 \text{ cm}^2/\text{sec}\) (see CETN II-4)]
SOLUTION:

a) calculate $L_0$ (metric units)

$$L_0 = 1.56T^2 - 1.56(10)^2 = 156 \text{ m}$$

b) calculate $w$ (see Eq. 4-8 in the SPM and CETN II-4)

$$w = [(\rho_s/\rho - 1)g]^{0.7} d_{50}^{-1.1}/[6^0.4]$$

[A] $w = [(1.59)\cdot 981]^{0.7} (0.02)^{1.1}/[6\cdot (0.01)^{0.4}]$

= 2.4 \text{ cm/sec} \quad (= 0.024 \text{ m/sec})

[B] $w = [(1.59)\cdot 981]^{0.7} (0.04)^{1.1}/[6\cdot (0.01)^{0.4}]$

= 5.2 \text{ cm/sec} \quad (= 0.052 \text{ m/sec})

c) evaluate relationships for each criterion

$$H_0/L_0 = 1/156 = 0.006$$

Criterion 1:

[A] $0.00070 (H_0/wT)^3 = 0.00070 \cdot (1/[0.024\cdot 10])^3 = 0.051$

0.006 < 0.051, \hspace{0.5cm} \therefore \text{erosion}

[B] $0.00070 (H_0/wT)^3 = 0.00070 \cdot (1/[0.052\cdot 10])^3 = 0.005$

0.006 > 0.005, \hspace{0.5cm} \therefore \text{accretion}$

Criterion 2:

[A] $4.8\cdot 10^8 (H_0/d_{50})^{-3.05} = 4.8\cdot 10^8 (1/0.0002)^{-3.05} = 0.002$

0.006 > 0.002, \hspace{0.5cm} \therefore \text{erosion}

[B] $4.8\cdot 10^8 (H_0/d_{50})^{-3.05} = 4.8\cdot 10^8 (1/0.0004)^{-3.05} = 0.021$

0.006 < 0.021, \hspace{0.5cm} \therefore \text{accretion}$

Criterion 3:

[A] $115 (\pi w/gT)^{1.5} = 115 \cdot ([3.14\cdot 0.024]/[9.81\cdot 10])^{1.5} = 0.002$

0.006 > 0.002, \hspace{0.5cm} \therefore \text{erosion}

[B] $115 (\pi w/gT)^{1.5} = 115 \cdot ([3.14\cdot 0.052]/[9.81\cdot 10])^{1.5} = 0.008$

0.006 < 0.008, \hspace{0.5cm} \therefore \text{accretion}$
All three criteria have indicated that the finer sand size beach will erode and the coarser sand beach will accrete under the given wave condition. In rare instances the criteria may give conflicting indications of erosion and accretion. If this occurs, assume no change from the existing state.

For completeness, using the same wave characteristics and a beach slope of 1/50, the Sunamura and Horikawa criterion (Eq. 1) predicts erosion for the 0.2-mm sand size and an "undetermined" profile shape for the 0.4-mm sand beach.

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REFERENCES:


